Delving into China's Government Subsidy for Digital Products: From Eliminating Purchase Caps to Maximizing Consumer Utility

Abstract

This essay investigates how policy interventions, specifically government subsidies and purchase limits, shape consumer choices between two selected models – iPhone 17 and Huawei Mate 70. A CES utility framework is used to model consumer preferences, and optimal bundles under three policy scenarios are derived using Kuhn-Tucker conditions (Baseline) and Lagrangian Method with Bordered Hessian verification (Shock 1 and Shock 2). The analysis shows that purchase limits impose an implicit cost preventing consumers from achieving the optimum. By removing the quantity restriction, consumers can fully express their preferences. Additionally, when subsidies operate without quantity limits, their impact becomes highly sensitive to substitutability: a higher ρ enlarges the magnitude of the shift toward the preferred brand, producing a 'winner-takes-most' outcome that extremely favors Apple Inc. These findings highlight how the mixture of subsidies and quantity restrictions can influence both consumer welfare and firms' performance in the market.

Keywords: CES Utility Function; Subsidy Policy; Purchase Limit; China Smartphone Market; Consumer Theory

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1. Introduction

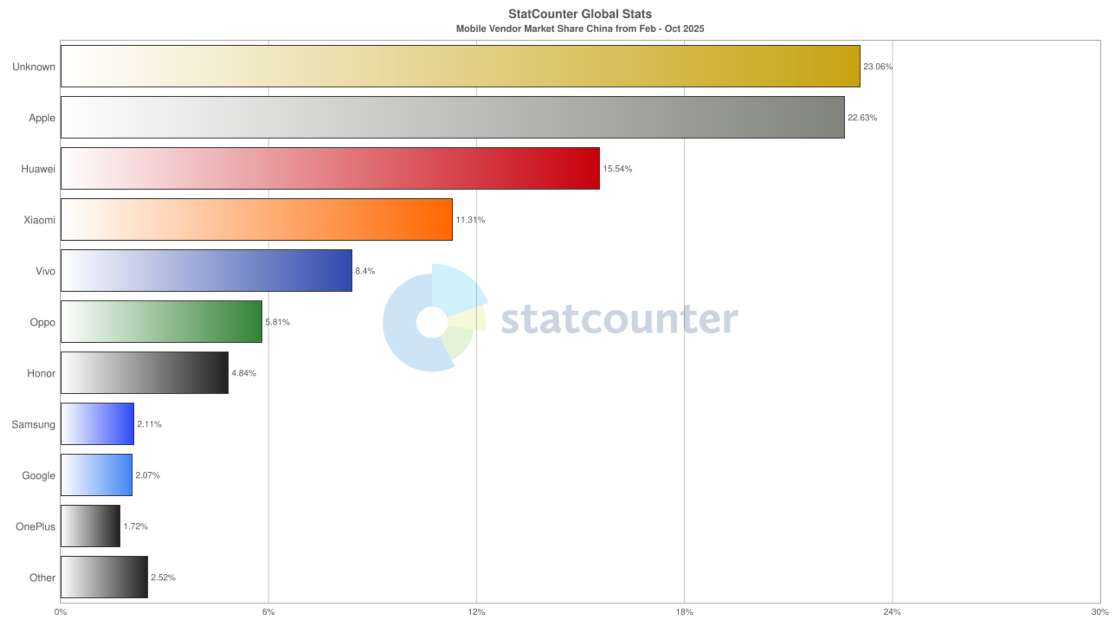
1.1 Background and Problem Statement

The landscape of the Chinese digital products market has been influenced by the intensifying Sino-US geopolitical competition and the ongoing trade war. This macro-environment has profoundly impacted market dynamics, especially for domestic firms like Huawei, which have faced significant restrictions on access to critical technologies and supply chains (Rasheed & Rasheed, 2022). Consequently, the pursuit of technological self-sufficiency and the promotion of domestic market champions have become central government strategic priorities. In short, the intensification of the trade war has exacerbated China's need for economic transformation and prompted the government to more resolutely adopt tools for stimulating domestic demand.

In this context, consumer subsidies — traditionally a tool for market regulation and social welfare — are also being developed by China to support the domestic digital ecosystem. Yi (2025) reported that, starting from January 20th, individual consumers purchasing new mobile phones, tablets, and smart watches priced under 6,000 yuan can receive a 15 percent subsidy of the product’s sales price, with 500 yuan per item at maximum. This policy applies to both domestic and foreign brands, with a quantity limit on subsidized items, representing a significant intervention in the digital products market.

Figure 1

Mobile Vendor Market Share China from Feb-Oct 2025 (StatCounter Global Stats, n.d.)



After the subsidy policy is implemented, Apple (22.63%) and Huawei (15.54%) emerge as the two dominant identified brands, as shown in Figure 1. Although unknown brands hold the largest market share (23.06%), this category aggregates data from numerous small or untracked vendors. To avoid introducing noise generated by heterogeneous or unfamiliar labels, the present analysis focuses on Apple and Huawei, the only brands with both substantial scale and stable consumer awareness.

Apple and Huawei's competitive structure aligns with historical trends in mobile telecommunications: multinational corporations have leveraged their technological advantages and intellectual property to secure market dominance, while firms from developing nations have faced significant barriers in establishing their competitive brands and gaining market entry (Zhu et al., 2006). Apple is a successful example of a multinational corporation, capturing a substantial share of China's market by leveraging its brand effect and delivering a premium user experience. In contrast, Huawei's recent ascent has been driven by rapid innovation, strategic alignment with national priorities, and increasing consumer identification with narratives of technological autonomy.

A comparison between Apple and Huawei is particularly instructive. Both brands share key similarities: commensurate price premiums, targeting consumers with comparable income levels, and incorporating cutting-edge chipsets and imaging systems. However, Rasheed and Rasheed (2022) stated that they embody divergent consumer identities: Apple appeals to consumers seeking a global and integrated ecosystem, whereas Huawei resonates with users who identify with technological autonomy against geopolitical pressures. Therefore, although the products are close substitutes in technical terms, consumer utilities are shaped by heterogeneous brand identities, loyalty drivers, and patriotic consumption sentiments.

This scenario creates an ideal setting to study the differential marginal utility of a price subsidy for heterogeneous consumer groups. While the subsidy aims to boost consumption and support domestic industry, its uniform ad valorem structure may introduce distortions (Fan & Zhang, 2022). Because the subsidy is proportional to price, its marginal benefit varies across consumers depending on brand preference intensity and willingness to substitute between foreign and domestic products. In markets where goods are technologically substitutable but emotionally and symbolically differentiated, a uniform subsidy may shift consumption allocation in ways that do not maximize aggregate welfare. Thus, the underlying economic problem is therefore to determine whether the current subsidy scheme generates an efficient consumption equilibrium and corresponding utility. This is a real-world policy issue with direct implications for subsidy design and the competitive balance between domestic and foreign firms.

1.2 Objective of the Study

This study aims to evaluate the welfare implications of China's digital product subsidy policy and explore pathways for its optimization. To this end, we pursue the following objectives:

To assess the welfare implications and inform the optimization of China's digital product subsidy policy, this study first proposes a consumer choice model centered on the two dominant brands. We explore the optimal consumption bundle under a utility maximization framework in the short term, where consumer decisions are constrained by income and a purchase quota.

Then, the influence of the uniform subsidy rate on market demand structure in the long term is studied through a comparative statics analysis, and the dynamic welfare analysis is carried out. The consumption equilibrium, determined by the interplay of brand preferences, prices, and the subsidy, reaches a stable state in the long run, which reveals the differential impact of the policy on heterogeneous consumer groups. To better predict consumer responses, the Constant Elasticity of Substitution (CES) utility function is also introduced to quantify the substitutability between foreign and domestic brands.

Finally, we calibrate the model with real-world market data and analyze the key outcomes through the economic frameworks used above to explore the potential efficiency gains of a more targeted subsidy scheme to enhance overall consumer welfare.

2. Mathematical Formulation

2.1 CES Utility Function

The Constant Elasticity of Substitution (CES) function, introduced by Arrow et al. (1961), is a core tool in macroeconomics for illustrating the substitutability between capital and labor. In consumer theory, the CES function effectively quantifies substitution patterns among differentiated goods (Varian, 1992; Tohamy & Mixon, 2004). In this essay, we present the CES utility as follows:

$$U(x_1, x_2) = A(\alpha {x_1}^{\rho} + \beta {x_2}^{\rho})^{\frac{1}{\rho}}$$

The function captures the utility relationship between two goods for consumers, with a constant elasticity of substitution between the goods. x_1 and x_2 denote the quantities of goods, and A represents the utility scale parameter. α represents the preference weight associated with good 1 (with β assigned to good 2), while $\rho = \frac{\sigma-1}{\sigma}$ links to the elasticity of substitution σ. A larger σ indicates that two goods are more likely to be substitutes for each other.

2.2 Assumptions

To simplify the study and construction of the economic model, our analysis relies on several key assumptions.

(1) We consider a representative consumer who chooses quantities x_1, x_2 (which are non-negative) to

maximize utility $U(x_1, x_2)$. Quantities are in the form of 'units' and we treat them as continuous.

(2) We assume Apple and Huawei are two oligopoly firms in the mobile phone industry in China.[1]

(3) Only one of the representative models from each company is considered in this essay, with Huawei Mate 70 and Apple iPhone 17.

(4) Prices of the two models are exogenous and taken as given by the consumer. Neither Huawei nor Apple adjusts prices.

(5) We use the current market shares of two companies as a proxy to calibrate the preference weight α and β in the CES function.

(6) We assume that the government offers a uniform subsidy rate s applied directly to the original prices, with no maximum subsidy amount and price range restriction on the products.

(7) Consumer Income I is assumed to be fixed. We set I = 10,000 RMB, representing a stylized portion of an urban resident's annual disposable income[2] allocated to purchasing smartphones.

(8) The purchase limit is set at θ = 2 units to represent a modified restriction that allows consumers to fully express their preferences, while in reality, the limit is 1.

(9) We normalize the scale parameter A to 1, as it does not affect optimal choices or comparative statics in the CES utility framework.

2.3 Notations

Relevant notations are written in Table 1 below.

Table 1

Definitions of variables and parameters

Symbol	Definition	Unit
x_1	Purchases of iPhone 17	units
x_2	Purchases of Huawei Mate 70	units
P_{x_1}	Market Price of iPhone 17	RMB
P_{x_2}	Market Price of Mate 70	RMB
α	Preference weight for iPhone 17	—
β	Preference weight for Huawei Mate 70	—
I	Stylized consumer income	RMB
s	Subsidy rate	%
θ	Maximum allowable purchase quantity under the policy	units
σ	Elasticity of substitution between the two models	—
ρ	Substitution parameter in the CES utility function	—

Note. α and β are calibrated using market share data.

[1] As shortly discussed in introduction, according to StatCounter (https://gs.statcounter.com/vendor-market-share/mobile/china), Firms with the biggest market share in China are Unknown (23.06%), Apple (22.63%), and Huawei (15.54%), where 'Unknown' refers to miscellaneous small firms in China. For simplicity, we ignore 'Unknown'.

[2] According to the National Bureau of Statistics (https://data.stats.gov.cn/), per capita disposable income of urban residents in 2024 is 54,188 RMB, we stylized the I in the model to be 10,000 RMB, allowing the model to capture realistic trade-offs.

2.4 Constraints under Three Scenarios

Our group designed three scenarios to capture the key institutional features of China's product subsidy policy and reflect the impact of subsidies and purchase restrictions on consumer behavior. All the scenarios have the same utility function below but with different constraints.

$$\max_{\{x_1, x_2\}} U(x_1, x_2) = \left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}}$$

Scenario 1 (Baseline: Subsidy + Purchase Limit) Constraint:

$$s.t.\ x_1 + x_2 \leq \theta \qquad (1 - s)P_{x_1}x_1 + (1 - s)P_{x_2}x_2 \leq I$$

This scenario demonstrates that the government, while providing subsidies, ensures the fair distribution of subsidy resources through quantity restrictions.

Scenario 2 (Eliminate Purchase Limit: Subsidy only) Constraint:

$$s.t.(1 - s)P_{x_1}x_1 + (1 - s)P_{x_2}x_2 \leq I$$

This scenario isolates the pure price effect of the subsidy by removing quantity restrictions.

Scenario 3 (Remove Subsidy) Constraint:

$$s.t. P_{x_1}x_1 + P_{x_2}x_2 \leq I$$

This scenario is used for comparative analysis of the impact of canceling subsidy policies on the market and consumers.

3. Solution & Theoretical Analysis

3.1 Baseline Scenario

With the aim of simulating a real-world condition, we begin by constructing a CES function along with two constraint functions, which correspond to the purchase quantity limit and budget constraint, respectively.

$$\max_{\{x_1, x_2\}} U(x_1, x_2) = \left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}}$$

$$s.t.\ x_1 + x_2 \leq \theta \qquad (1 - s)P_{x_1}x_1 + (1 - s)P_{x_2}x_2 \leq I$$

To figure out the optimization problem of the CES utility function, we adopt the Kuhn-Tucker Conditions given that the constraints specified herein are inequality constraints.

<u>Step 1: Construct the Lagrangian</u>

$$Z = \left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}} + \lambda_1(\theta - x_1 - x_2) + \lambda_2[I - (1-s)P_{x_1}x_1 - (1-s)P_{x_2}x_2]$$

Step 2: First-Order Partial Derivatives

$$Z_{x_1} = \frac{\partial Z}{\partial x_1} = \alpha x_1^{\rho-1}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-1} - \lambda_1 - P_{x_1}\lambda_2(1-s) \quad (1)$$

$$Z_{x_2} = \frac{\partial Z}{\partial x_2} = \beta x_2^{\rho-1}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-1} - \lambda_1 - P_{x_2}\lambda_2(1-s) \quad (2)$$

$$Z_{\lambda_1} = \frac{\partial Z}{\partial \lambda_1} = \theta - x_1 - x_2 \quad (3)$$

$$Z_{\lambda_2} = \frac{\partial Z}{\partial \lambda_2} = I - P_{x_1}x_1(1-s) - P_{x_2}x_2(1-s) \quad (4)$$

Step 3: Kuhn-Tucker Conditions:

$$\begin{cases} Z_{x_1} = \alpha x_1^{\rho-1}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-1} - \lambda_1 - P_{x_1}\lambda_2(1-s) \leq 0 \\ x_1 \geq 0 \\ x_1 Z_{x_1} = 0 \end{cases}$$

$$\begin{cases} Z_{x_2} = \beta x_2^{\rho-1}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-1} - \lambda_1 - P_{x_2}\lambda_2(1-s) \leq 0 \\ x_2 \geq 0 \\ x_2 Z_{x_2} = 0 \end{cases}$$

$$\begin{cases} Z_{\lambda_1} = \theta - x_1 - x_2 \geq 0 \\ \lambda_1 \geq 0 \\ \lambda_1 Z_{\lambda_1} = 0 \end{cases}$$

$$\begin{cases} Z_{\lambda_2} = I - P_{x_1}x_1(1-s) - P_{x_2}x_2(1-s) \geq 0 \\ \lambda_2 \geq 0 \\ \lambda_2 Z_{\lambda_2} = 0 \end{cases}$$

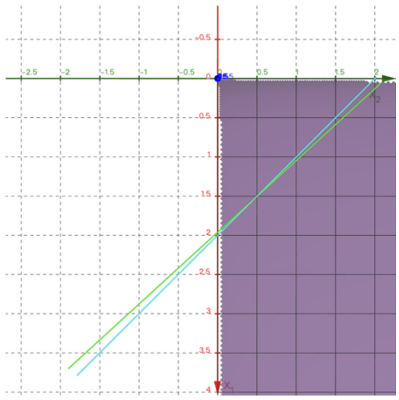
Step 4: Solve Kuhn-Tucker Conditions:

As mentioned in our previous section, x_1 and x_2 represent the number of iPhones and Huawei phones consumed by customers respectively, as a result, both x_1 and x_2 are non-negative. Moreover, we want to investigate into utility maximization through CES functions, $x_1 = 0$ and $x_2 = 0$ will lead to zero utility, which will not be an optimum apparently. In short, here we only consider the case when $x_1 > 0$ and $x_2 > 0$.

As for the constraints, based on the real data, we obtained the figures next page (Figure 2), which is a top-down view of the graph. The purple shaded surface represents the CES utility function, the light blue plane stands for the purchasing quantity limit while the light green plane is the budget constraint.

Figure 2

Top-down view of the constructed model



As depicted in Figure 2, both the purchasing quantity limit and the budget constraint are binding. As a result, we will discuss the circumstance when $\lambda_1 > 0$ and $\lambda_2 > 0$. The intersection of both constraint functions represents the optimal value.

As mentioned above, we only consider the case when $x_1 > 0$ and $x_2 > 0$. To satisfy complementary slackness condition $x_1 Z_{x_1} = 0$ and $x_2 Z_{x_2} = 0$, we need to let $Z_{x_1} = 0$; $Z_{x_2} = 0$.

$$Z_{x_1} = \alpha x_1^{\rho-1}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-1} - \lambda_1 - P_{x_1}\lambda_2(1-s) = 0$$

$$x_1 Z_{x_1} = 0$$

$$Z_{x_2} = \beta x_2^{\rho-1}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-1} - \lambda_1 - P_{x_2}\lambda_2(1-s) = 0$$

$$x_2 Z_{x_2} = 0$$

For the Lagrange multipliers, we consider $\lambda_1 > 0$, $\lambda_2 > 0$. By the complementary slackness condition, this requires letting $Z_{\lambda_1} = 0$ and $Z_{\lambda_2} = 0$.

$$Z_{\lambda_1} = \theta - x_1 - x_2 = 0$$

$$\lambda_1 Z_{\lambda_1} = 0$$

$$Z_{\lambda_2} = I - P_{x_1}x_1(1-s) - P_{x_2}x_2(1-s) = 0$$

$$\lambda_2 Z_{\lambda_2} = 0$$

As we know, if both constraint functions are binding, the optimal point must satisfy both constraints and fall on both functions. As a result, the optimal point corresponds to the intersection between the two constraint functions. To derive this solution, we solve the system of equations formed by (3) and (4) by setting both equal to zero and compute the values of variables that satisfy both equations.

$$Z_{\lambda_1} = \theta - x_1 - x_2 = 0 \quad (5a)$$

$$Z_{\lambda_2} = I - P_{x_1}x_1(1-s) - P_{x_2}x_2(1-s) = 0 \quad (5b)$$

From (5a) we get:

$$x_1 = \theta - x_2$$

Substitute $x_1 = \theta - x_2$ back to (5b), we get:

$$I - P_{x_1}(\theta - x_2)(1-s) - P_{x_2}x_2(1-s) = 0$$

$$I - (1-s)\left(P_{x_1}\theta - P_{x_1}x_2 + P_{x_2}x_2\right) = 0$$

$$I - (1-s)P_{x_1}\theta = -x_2(1-s)(P_{x_1} - P_{x_2})$$

$$x_2^* = \frac{(1-s)P_{x_1}\theta - I}{(1-s)\left(P_{x_1} - P_{x_2}\right)}$$

Substitute x_2^* back to $x_1 = \theta - x_2$, we can get

$$x_1^* = \theta - x_2^* = \frac{I - (1-s)P_{x_2}\theta}{(1-s)(P_{x_1} - P_{x_2})}$$

Finally, regarding the CES utility, we obtain its maximization by substituting both x_1^* and x_2^*:

$$U(x_1^*, x_2^*) = \left\{\alpha\left[\frac{I - (1-s)P_{x_2}\theta}{(1-s)(P_{x_1} - P_{x_2})}\right]^{\rho} + \beta\left[\frac{(1-s)P_{x_1}\theta - I}{(1-s)\left(P_{x_1} - P_{x_2}\right)}\right]^{\rho}\right\}^{\frac{1}{\rho}}$$

3.2 Policy Shock 1: Eliminating Purchase Limit

Having established the optimal consumption bundle of baseline scenario with both purchase limit and budget constraint. In this section, we analyze the Policy Shock 1, which is similar to the baseline model except that purchase limit is eliminated:

$$\max_{\{x_1, x_2\}} U(x_1, x_2) = (\alpha x_1^{\rho} + \beta x_2^{\rho})^{\frac{1}{\rho}}$$

$$s.t.\ (1-s)P_{x_1}x_1 + (1-s)P_{x_2}x_2 \le I$$

Varian (2014) stated that if utility is strictly monotonic, whole budget will be spent by the representative consumer. The optimal consumption bundle is found on the budget line, with no income left over:

$$MU_{x_1} = \alpha x_1^{\rho-1}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-1} > 0, \qquad since\ \alpha, \beta, \rho > 0$$

$$MU_{x_2} = \beta x_2^{\rho-1}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-1} > 0, \qquad since\ \alpha, \beta, \rho > 0$$

Given the marginal utility functions are positive, indicating that the utility function is monotonically increasing. Thus, we directly set the budget constraint equal to income I and get the function $g_{(x_1, x_2)}$. To compute the optimal quantity x_1^* and x_2^* that maximizes the consumer's utility, we establish the Lagrangian function:

$$g_{(x_1, x_2)} = I - (1-s)\left(P_{x_1}x_1 + P_{x_2}x_2\right)$$

$$\mathcal{L}(x_1, x_2, \lambda) = \left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}} + \lambda\left[I - (1-s)\left(P_{x_1}x_1 + P_{x_2}x_2\right)\right]$$

where λ is the Lagrange multiplier associated with the budget constraint, which measures how optimal value change if constraint changes.

The first-order necessary conditions (FONCs) for a maximum are obtained by taking the partial derivatives and setting them to zero:

$$\begin{cases} \dfrac{\partial \mathcal{L}}{\partial x_1} = \alpha x_1^{\rho-1}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-1} - \lambda(1-s)P_{x_1} = 0 & (6) \\ \dfrac{\partial \mathcal{L}}{\partial x_2} = \beta x_2^{\rho-1}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-1} - \lambda(1-s)P_{x_2} = 0 & (7) \\ \dfrac{\partial \mathcal{L}}{\partial \lambda} = I - (1-s)\left(P_{x_1}x_1 + P_{x_2}x_2\right) = 0 & (8) \end{cases}$$

From the first-order conditions (6) and (7), eliminating the Lagrange multiplier λ gives:

(6) divided by (7)

$$\frac{\alpha}{\beta}\left(\frac{x_1}{x_2}\right)^{\rho-1} = \frac{P_{x_1}}{P_{x_2}}$$

$$x_1 = \left(\frac{\beta P_{x_1}}{\alpha P_{x_2}}\right)^{\frac{1}{\rho-1}} x_2$$

Substitute to budget constraint:

$$(1-s)P_{x_1}x_1 + (1-s)P_{x_2}x_2 = I$$

$$(1-s)P_{x_1}\left(\frac{\beta P_{x_1}}{\alpha P_{x_2}}\right)^{\frac{1}{\rho-1}} x_2 + (1-s)P_{x_2}\, x_2 = I$$

Obtain x_1^* and x_2^* :

$$\begin{cases} x_1^* = \dfrac{\dfrac{I}{1-s}}{P_{x_1} + P_{x_2}\left(\dfrac{\beta P_{x_1}}{\alpha P_{x_2}}\right)^{\frac{1}{1-\rho}}} \\ x_2^* = \dfrac{\dfrac{I}{1-s}}{P_{x_2} + P_{x_1}\left(\dfrac{\beta P_{x_1}}{\alpha P_{x_2}}\right)^{\frac{1}{\rho-1}}} \end{cases}$$

The first-order necessary conditions (FONCs) are used to identify all the stationary point. To control for the effect of saddle point and local minimum point, constructing second-order conditions by examining the Bordered Hessian matrix:

$$\bar{H} = \begin{pmatrix} 0 & g_{x_1} & g_{x_2} \\ g_{x_1} & U_{x_1x_1} & U_{x_1x_2} \\ g_{x_2} & U_{x_2x_1} & U_{x_2x_2} \end{pmatrix}$$

Second order derivative of utility function:

$$U_{x_1x_1} = \alpha\beta(\rho-1)\, x_1^{\rho-2}\, x_2^{\rho}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{-\frac{2\rho-1}{\rho}}$$

$$U_{x_2x_2} = \alpha\beta(\rho-1)\, x_1^{\rho}\, x_2^{\rho-2}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{-\frac{2\rho-1}{\rho}}$$

$$U_{x_1x_2} = U_{x_2x_1} = -\alpha\beta(\rho-1)\, x_1^{\rho-1}\, x_2^{\rho-1}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{-\frac{2\rho-1}{\rho}}$$

Bordered Hessian matrix as follow:

$$\overline{H} = \begin{pmatrix} 0 & g_{x_1} & g_{x_2} \\ g_{x_1} & U_{x_1x_1} & U_{x_1x_2} \\ g_{x_2} & U_{x_2x_1} & U_{x_2x_2} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -P_{x_1}(1-s) & -P_{x_2}(1-s) \\ -P_{x_1}(1-s) & \alpha\beta(\rho-1)\, x_1^{\rho-2}\, x_2^{\rho}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{-\frac{2\rho-1}{\rho}} & -\alpha\beta(\rho-1)\, x_1^{\rho-1}\, x_2^{\rho-1}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{-\frac{2\rho-1}{\rho}} \\ -P_{x_2}(1-s) & -\alpha\beta(\rho-1)\, x_1^{\rho-1}\, x_2^{\rho-1}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{-\frac{2\rho-1}{\rho}} & \alpha\beta(\rho-1)\, x_1^{\rho}\, x_2^{\rho-2}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{-\frac{2\rho-1}{\rho}} \end{pmatrix}$$

Determinant of a matrix $\overline{H}$ is given below:

$$|\overline{H}| = -\frac{\alpha\beta x_1^{\rho} x_2^{\rho}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{1/\rho}(\rho-1)(s-1)^2\left(P_{x_1}^2 x_1^2 + 2P_{x_1}P_{x_2}x_1x_2 + P_{x_2}^2 x_2^2\right)}{x_1^2 x_2^2\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^2}$$

$$since\ \alpha, \beta > 0, \rho < 1, \qquad |\overline{H}| > 0$$

For a constrained optimization problem analysis, the positive determinant of Bordered Hessian matrix ensures the local maximum point under the given constraint.

3.3 Policy Shock 2: Removing Subsidy

Following an analysis of the scenario of relaxed purchase restrictions, we further examine the impacts of completely eliminating government subsidies on optimal consumption decisions. In the absence of subsidies, consumers are required to bear the full pre-subsidy product prices without any financial relief, leading to an adjusted budget constraint. All other assumptions remain unchanged and consistent with those of the baseline model.

$$\max_{\{x_1, x_2\}} U(x_1, x_2) = (\alpha x_1^{\rho} + \beta x_2^{\rho})^{\frac{1}{\rho}}$$

$$s.t.\ P_{x_1}x_1 + P_{x_2}x_2 \le I$$

Similar to Chapter 3.2, under monotonic preferences, the optimal consumption bundle must lie on the budget line (i.e., the constraint is binding), with no unspent income. So, we directly construct the Lagrange function for this constrained optimization problem as follows, where λ denotes the Lagrange multiplier corresponding to the budget constraint:

$$\mathcal{L}(x_1, x_2, \lambda) = (\alpha x_1^\rho + \beta x_2^\rho)^{\frac{1}{\rho}} + \lambda(I - P_{x_1}x_1 - P_{x_2}x_2)$$

To determine the local maximum, we first compute the first-order partial derivatives. Based on the first-order conditions, the system of simultaneous equations is then solved as follows:

$$\begin{cases} \mathcal{L}_{x_1} = \alpha x_1^{\rho-1}\left(\alpha x_1^\rho + \beta x_2^\rho\right)^{\frac{1}{\rho}-1} - \lambda P_{x_1} = 0 \\ \mathcal{L}_{x_2} = \beta x_2^{\rho-1}\left(\alpha x_1^\rho + \beta x_2^\rho\right)^{\frac{1}{\rho}-1} - \lambda P_{x_2} = 0 \\ \mathcal{L}_\lambda = I - P_{x_1}x_1 - P_{x_2}x_2 = 0 \end{cases}$$

Utilizing the same calculation methodology as presented in 3.2, we can acquire the desired results:

$$\begin{cases} x_1^* = \dfrac{I}{P_{x_1} + P_{x_2}(\frac{P_{x_1}\beta}{P_{x_2}\alpha})^{\frac{1}{1-\rho}}} \\ x_2^* = \dfrac{I}{P_{x_1}(\frac{P_{x_2}\alpha}{P_{x_1}\beta})^{\frac{1}{1-\rho}} + P_{x_2}} \end{cases}$$

And

$$U^*(x_1, x_2) = \left[\alpha\left(\frac{I}{P_{x_1}+P_{x_2}\left(\frac{P_{x_1}\beta}{P_{x_2}\alpha}\right)^{\frac{1}{1-\rho}}}\right)^\rho + \beta\left(\frac{I}{P_{x_1}(\frac{P_{x_2}\alpha}{P_{x_1}\beta})^{\frac{1}{1-\rho}}+P_{x_2}}\right)^\rho\right]^{\frac{1}{\rho}}$$

Finally, we use Bordered Hessian matrix to verify the constraint maximum, we define $g(x_1, x_2) = I - P_{x_1}x_1 - P_{x_2}x_2$. In our model, the Bordered Hessian is:

$$\bar{H} = \begin{pmatrix} 0 & g_{x_1} & g_{x_2} \\ g_{x_1} & U_{x_1x_1} & U_{x_1x_2} \\ g_{x_2} & U_{x_2x_1} & U_{x_2x_2} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -P_{x_1} & -P_{x_2} \\ -P_{x_1} & -\alpha\beta x_1^{\rho-2}x_2^\rho(1-\rho)\left(\alpha x_1^\rho + \beta x_2^\rho\right)^{\frac{1}{\rho}-2} & \alpha\beta x_1^{\rho-1}x_2^{\rho-1}(1-\rho)\left(\alpha x_1^\rho + \beta x_2^\rho\right)^{\frac{1}{\rho}-2} \\ -P_{x_2} & \alpha\beta x_1^{\rho-1}x_2^{\rho-1}(1-\rho)\left(\alpha x_1^\rho + \beta x_2^\rho\right)^{\frac{1}{\rho}-2} & -\alpha\beta x_1^\rho x_2^{\rho-2}(1-\rho)\left(\alpha x_1^\rho + \beta x_2^\rho\right)^{\frac{1}{\rho}-2} \end{pmatrix}$$

$$|\bar{H}| = (-1)^{1+1} \times 0 \begin{vmatrix} -\alpha\beta x_1^{\rho-2} x_2^{\rho}(1-\rho)\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-2} & \alpha\beta x_1^{\rho-1} x_2^{\rho-1}(1-\rho)\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-2} \\ \alpha\beta x_1^{\rho-1} x_2^{\rho-1}(1-\rho)\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-2} & -\alpha\beta x_1^{\rho} x_2^{\rho-2}(1-\rho)\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-2} \end{vmatrix}$$

$$+ (-1)^{1+2}(-P_{x_1}) \begin{vmatrix} -P_{x_1} & \alpha\beta x_1^{\rho-1} x_2^{\rho-1}(1-\rho)\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-2} \\ -P_{x_2} & -\alpha\beta x_1^{\rho} x_2^{\rho-2}(1-\rho)\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-2} \end{vmatrix}$$

$$+ (-1)^{1+3}(-P_{x_2}) \begin{vmatrix} -P_{x_1} & -\alpha\beta x_1^{\rho-2} x_2^{\rho}(1-\rho)\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-2} \\ -P_{x_2} & \alpha\beta x_1^{\rho-1} x_2^{\rho-1}(1-\rho)\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-2} \end{vmatrix}$$

Through rigorous calculation and verification, we can draw the conclusion that

$$|\bar{H}| = \alpha\beta x_1^{\rho-2} x_2^{\rho-2}(1-\rho)\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}-2}(P_{x_1}^2 x_1^2 + P_{x_2}^2 x_2^2) > 0$$

That means we can conclude that the total utility will be maximized when the economic purchasing quantity of each type of phone x_1^* and x_2^* is adopted.

3.4 Comparative Statics

We now conduct comparative statics to examine how the optimal consumption bundle (x_1^*, x_2^*) respond to changes in some key parameters: prices P_{x_1}, P_{x_2}, CES elasticity ρ, subsidy rate s, and purchase limit θ.

3.4.1 Price Effect

For the optimal bundle, it has:

$$MRS = -\frac{MU_{x_1}}{MU_{x_2}} = -\frac{P_{x_1}}{P_{x_2}} = -price\ ratio$$

$$-\frac{\alpha x_1^{\rho-1}(\alpha x_1^{\rho} + \beta x_2^{\rho})^{\frac{1}{\rho}-1}}{\beta x_2^{\rho-1}(\alpha x_1^{\rho} + \beta x_2^{\rho})^{\frac{1}{\rho}-1}} = -\frac{P_{x_1}}{P_{x_2}}$$

$$\frac{\alpha}{\beta}\left(\frac{x_1}{x_2}\right)^{\rho-1} = \frac{P_{x_1}}{P_{x_2}}$$

$$\frac{x_1}{x_2} = \left(\frac{\beta}{\alpha}\frac{P_{x_1}}{P_{x_2}}\right)^{\frac{1}{\rho-1}}$$

Thus, at the optimal point,

$$\frac{x_1^*}{x_2^*} = \left(\frac{\beta}{\alpha}\frac{P_{x_1}}{P_{x_2}}\right)^{\frac{1}{\rho-1}}$$

We have the partial derivatives solved by Mathematica:

$$\frac{\partial(\frac{x_1}{x_2})}{\partial P_{x_1}}=\frac{\beta(\frac{\beta P_{x_1}}{\alpha P_{x_2}})^{\frac{1}{\rho-1}-1}}{\alpha P_{x_2}(\rho-1)}<0$$

$$\frac{\partial(\frac{x_1}{x_2})}{\partial P_{x_2}}=-\frac{\beta P_{x_1}(\frac{\beta P_{x_1}}{\alpha P_{x_2}})^{\frac{1}{\rho-1}-1}}{\alpha P_{x_2}^2(\rho-1)}>0$$

This indicates that an increase in Apple's price will reduce the relative consumption of Apple products, while an increase in Huawei's price will increase Apple's relative consumption, which is consistent with standard demand theory.

3.4.2 Elasticity ρ Effect

The CES substitution parameter ρ influences substitutability:

$$\frac{\partial(\frac{x_1}{x_2})}{\partial \rho}=-\frac{(\frac{\beta P_{x_1}}{\alpha P_{x_2}})^{\frac{1}{\rho-1}}Log[\frac{\beta P_{x_1}}{\alpha P_{x_2}}]}{(\rho-1)^2}$$

The sign of $\frac{\partial(\frac{x_1}{x_2})}{\partial \rho}$ depends on goods' prices and consumer preferences.

While in our essay, combining with real-world data, $\frac{\partial(\frac{x_1}{x_2})}{\partial \rho}>0$, which means as ρ increases, relative consumption of Apple also increases. This phenomenon will be further discussed later.

3.4.3 Subsidy Effect

To analyze the effect of subsidy s on consumers' optimal bundles, we start from the theoretical bundles derived in Chapter 3.2 Policy Shock 1: Eliminate Purchase Limit, where a representative consumer's CES utility maximization under a budget constraint with government subsidy.

Recall $x_1^*=\frac{\frac{I}{1-s}}{P_{x_1}+P_{x_2}(\frac{P_{x_1}\beta}{P_{x_2}\alpha})^{\frac{1}{1-\rho}}}$; $x_2^*=\frac{\frac{I}{1-s}}{P_{x_1}(\frac{P_{x_2}\alpha}{P_{x_1}\beta})^{\frac{1}{1-\rho}}+P_{x_2}}$

Then

$$\frac{\partial x_1^*}{\partial s}=\frac{I}{(P_{x_1}+(\frac{\beta P_{x_1}}{\alpha P_{x_2}})^{\frac{1}{1-\rho}}P_{x_2})(-1+s)^2}>0$$

$$\frac{\partial x_2^*}{\partial s}=\frac{I}{(P_{x_2}+(\frac{\alpha P_{x_2}}{\beta P_{x_1}})^{\frac{1}{1-\rho}}P_{x_1})(-1+s)^2}>0$$

The positive signs indicate that an increase in the subsidy rate reduces effective prices, thereby raising the quantities of both goods purchased. This phenomenon demonstrates that subsidies serve as a direct policy tool to stimulate demand for both Apple and Huawei smartphones by lowering the effective prices.

3.4.4 Purchase Limit θ Effect

The effect of the purchase limit θ on the consumer's optimal bundle can be analyzed based on the utility. When the quantity restriction $x_1 + x_2 \leq \theta$ is binding, the consumer is forced to allocate purchases within the limit. Comparing the utility derived in later Chapter 4.2, an increase in θ relaxes the constraint, allowing the consumer to move closer to the optimal bundle, where, without the limit, increasing total utility, and vice versa.

Therefore, changes in θ directly affect the feasible range of consumption, with higher θ increasing the consumer's flexibility. This illustrates the policy effect of purchase limits on consumption decisions and highlights how regulatory constraints can cap consumer utility even if income and prices remain unchanged.

4. Empirical Analysis

4.1 Estimation of Parameter ρ

Theoretically, ρ is normally treated as an exogenous variable. However, given that we didn't have access to its exact value in the real world, we estimated it through systematic inference based on the research context and empirical data. The value of ρ in the remaining parts of the study will all be based on the results inferred here.

In our project, the parameter ρ is derived by simplifying relevant conditions, considering to maximize CES utility function $U(x_1, x_2) = \left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}}$ under the budget constraint $P_{x_1}x_1 + P_{x_2}x_2 \leq I$. Note that as illustrated in previous Chapter 3.2, the budget constraint will hold with equality when figuring out the optimal solution. The inference is as follows:

$$\max_{\{x_1, x_2\}} U(x_1, x_2) = \left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}}$$

$$s.t.\ P_{x_1}x_1 + P_{x_2}x_2 = I$$

According to Chapter 3.4.1, utility optimization condition holds when: $MRS = -Price\ Ratio$

Given this:

$$\frac{x_1}{x_2} = \left(\frac{\beta}{\alpha} \cdot \frac{P_{x_1}}{P_{x_2}}\right)^{\frac{1}{\rho - 1}}$$

Take the natural logarithm of both sides of the equation:

$$\ln\left(\frac{x_1}{x_2}\right) = \frac{1}{\rho - 1}\ln(\frac{\beta}{\alpha}\cdot\frac{P_{x_1}}{P_{x_2}})$$

$$\therefore \rho = 1 + \frac{\ln(\frac{\beta}{\alpha}\cdot\frac{P_{x_1}}{P_{x_2}})}{\ln\left(\frac{x_1}{x_2}\right)}$$

Based on the real-world sales data from Jingdong (https://www.jd.com/), we conducted relevant estimations and obtained $\rho \approx 0.8745$.

4.2 Optimal Bundle Under Each Scenario

As shown in Table 2 and Figure 3, the baseline allocation shows consumption level of x_1= 1.5334, and x_2= 0.4666, generating a utility of U = 1.0812. Under Shock 1, the consumer shift notably to iPhone (here x_1=1.7964), with Huawei drops to 0.1797, leading to a slightly higher utility (U = 1.0924). By contrast, under shock 2, the utility significantly decreases to 0.9285, with optimal bundle x_1= 1.5269, and x_2= 0.1527.

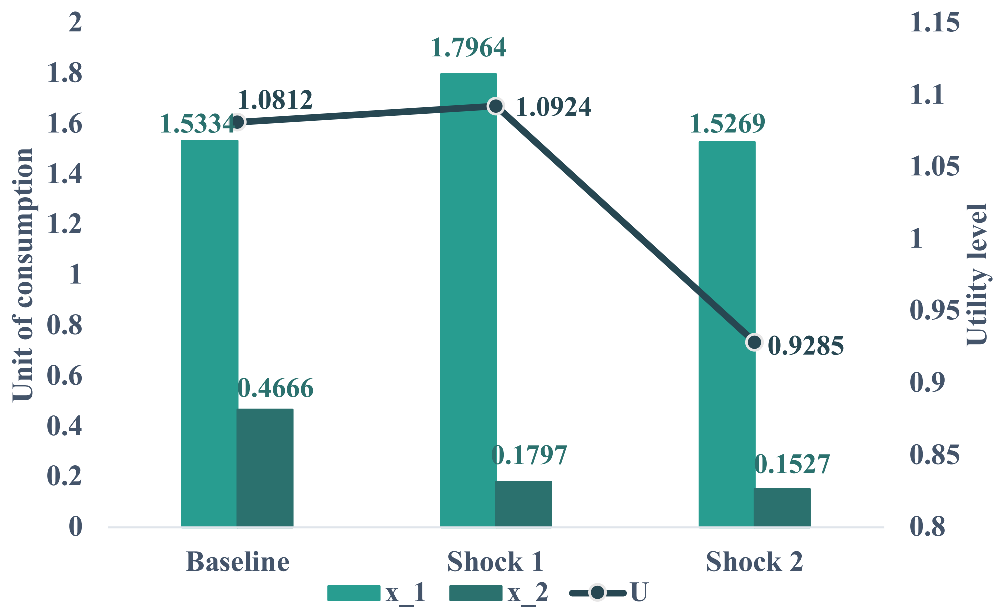
Table 2

Optimal Bundles and Utility in Each Scenario

	Baseline	Shock 1	Shock 2
x_1^*	1.5334	1.7964	1.5269
x_2^*	0.4666	0.1797	0.1527
$\frac{x_1^*}{x_2^*}$	3.2864	9.9973	9.9973
max U	1.0812	1.0924	0.9285

Figure 3

Comparison of Optimal Bundles and Utility under Each Scenario



4.3 Sensitivity Analysis

This section examines how the optimal bundles respond to different values of the substitution parameter ρ. Four parameter values are considered $-\rho$ = 0.4, 0.6, 0.8745, 0.9, representing increasing degrees of substitutability between two smartphone models. For each value, we recompute the optimal consumption bundles, relative quantities, and utilities under three scenarios, illustrated in Table 3 below.

Table 3

Optimal Consumption Bundles Across Different Values of ρ

Scenario	ρ	x_1^*	x_2^*	$\frac{x_1^*}{x_2^*}$	max U
Baseline	0.4	1.5334	0.4666	3.2864	1.0090
	0.6	1.5334	0.4666	3.2864	1.0402
	0.8745	1.5334	0.4666	3.2864	1.0812
	0.9	1.5334	0.4666	3.2864	1.0849
Shock 1	0.4	1.2520	0.7735	1.6186	1.0406
	0.6	1.3570	0.6590	2.0593	1.0489
	0.8745	1.7964	0.1797	9.9973	1.0924
	0.9	1.8660	0.1038	17.9826	1.1032
Shock 2	0.4	1.0642	0.6575	1.6186	0.8845
	0.6	1.1535	0.5601	2.0593	0.8916
	0.8745	1.5269	0.1527	9.9973	0.9285
	0.9	1.5861	0.0882	17.9826	0.9377

Under the baseline scenario, the optimal bundle remains identical across all values of ρ. This is because the solution is determined by the intersection of the Budget Constraint and the quantity restriction, so changes in ρ do not affect the bundles.

By contrast, in Shock 1 and 2, the optimal bundles vary with ρ. As ρ increases, the consumption of good 1 rises while the consumption of good 2 declines, and the maximum utility attained increases correspondingly. The ratio $\frac{x_1}{x_2}$ remains identical across two shocks under same ρ because the relative price $\frac{P_{x_1}}{P_{x_2}}$ is unchanged. However, as ρ increases, the ratio $\frac{x_1}{x_2}$ becomes larger, which is consistent with the comparative statics analysis in Chapter 3.4.2.

5. Economics Interpretation and Discussion

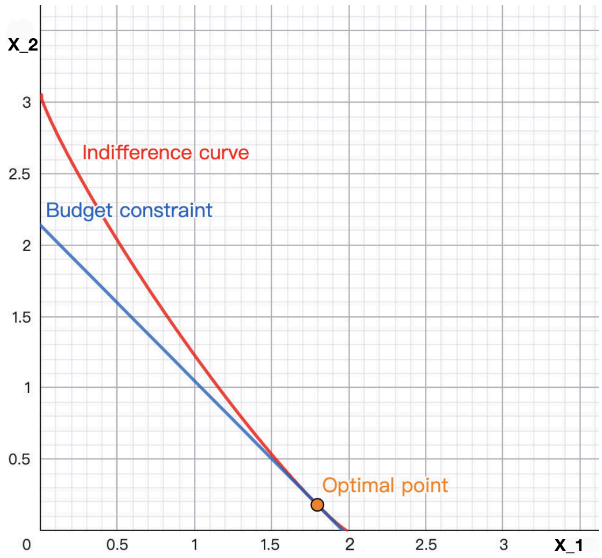
5.1 Effect of the Purchase Limit

Compared to the baseline scenario, utility under Shock 1 (where the purchase limit is removed) is higher, regardless of the value of ρ. In the baseline scenario, the purchase limit ($x_1 + x_2 \leq 2$) may impose an implicit cost on consumers, preventing them from reaching the standard interior optimum where $MRS = -Price\ ratio$. With both constraints binding, consumers are unable to adjust their bundles freely, the feasible set becomes artificially restricted, and the intersection of the two constraint functions becomes the optimal solution. The indifference curve of the consumer cannot be tangent to the budget constraint line, leading to suppressed utility at a lower level (U = 1.0812).

Under shock 1 scenario, removing the purchase limit enables consumers to regain the ability to express their inherent preferences, satisfying the condition where $MRS = -Price\ ratio$. Now, according to Figure 4, the consumption bundle revised from $(x_1 = 1.5334, x_2 = 0.4666)$ to $(x_1 = 1.7964, x_2 = 0.1797)$, with the indifference curve now tangent to the budget line.

Figure 4

Condition when Budget Constraint is tangent to Utility Function



The utility gaining from the removal of the limit reflects a reduction in allocative distortion, consumers can freely adjust their consumption bundles, which enables the achievement of maximum utility ($U = 1.0924$).

5.2 Substitutability and the Role of ρ

According to what we have investigated in Chapter 3.1 and Chapter 4.3, the optimal bundle of iPhones and

Huawei phones is independent of ρ in the baseline scenario. When comparing the optimal consumption bundle in two shock scenarios (where the purchase limit is removed), we observed that higher substitutability (i.e., high ρ) correlates with an evident reallocation of consumption toward the preferred brand (Apple).

Employing limit thinking, as ρ approaches 1, the CES function is reconstructed into $\lim_{\rho\to 1} U(x_1, x_2) = \lim_{\rho\to 1}\left(\alpha x_1^{\rho} + \beta x_2^{\rho}\right)^{\frac{1}{\rho}} = \alpha x_1 + \beta x_2$, which is exactly the utility function of two perfect substitutes. Under this circumstance, the elasticity of substitution σ: $\lim_{\rho\to 1} \sigma = \frac{1}{1-\rho} = \infty$, indicating that iPhones and Huawei phones are infinitely substitutable in terms of consumer preferences, consumers will be highly sensitive towards both goods. Thus, we consider in the perfect substitute conditions, $MRS = -\frac{MU_{x_1}}{MU_{x_2}} = -\frac{\alpha}{\beta} = -\frac{59.29\%}{40.71\%} \approx -1.456$, while the $price\ ratio = \frac{P_{x_1}}{P_{x_2}} = \frac{5999}{5499} \approx 1.091$. Apparently, $|MRS| > price\ ratio$. Rearranging the equation yields $\frac{MU_{x_1}}{MU_{x_2}} > \frac{P_{x_1}}{P_{x_2}}$, thus $\frac{MU_{x_1}}{P_{x_1}} > \frac{MU_{x_2}}{P_{x_2}}$, indicating that marginal utility per RMB for x_1 (iPhones) is higher than that for x_2 (Huawei phones). Thus, it is reasonable for consumers to allocate a larger amount of money to purchase iPhones given that the two products are highly substitutable.

This explains the explosive increase in the ratio $\frac{x_1}{x_2}$ (from 1.6186 to 17.9826 at $\rho = 0.9$), revealing that iPhones almost crowd out Huawei phones in the market. In such a condition, a symmetric subsidy could even disproportionately benefit the foreign brand Apple, which exhibits a stronger intrinsic attractiveness and preference among consumers. This insight reveals a potential structural problem that may be ignored before: the substitutability among certain goods might need more attention. Overlooking substitutability might unintentionally reinforce existing market dominance if preference asymmetries are strong, thus leading to misalignment between policy objectives and actual outcomes after implementation.

5.3 Implications on Consumption and Market Efficiency

When government subsidies for digital products are completely withdrawn, the aforementioned positive effects on consumption and welfare will be reversed, and profound impacts will be exerted on consumer behavior and market efficiency by restoring the pre-subsidy price system. Specifically, this price restoration reshapes consumer decision-making and generates far-reaching impacts on both consumption behavior and market efficiency. From a price perspective, ending subsidies directly eliminates consumers' prior financial relief and pushes up the effective prices of Apple and Huawei phones. This price hike triggers an income effect, as higher effective prices reduce real purchasing power and tighten consumers' budget constraints, which may ultimately lower demand for mobile devices. Notably, the model's optimal consumption mix reflects this balance as consumers prioritize utility-aligned products amid higher prices and ultimately make rational adjustments to purchase quantities.

Furthermore, turning to market efficiency, removing subsidies eliminates policy-induced price distortions and strengthens the market's role in resource allocation. Adverse consequences from government subsidies can arise, especially when subsidies lead to the inefficient phenomenon of improper resource allocation (Song et al., 2022). Under subsidy policies, artificially lowered prices can obscure true supply-demand dynamics, often resulting in overconsumption or inefficient allocation of factors. However, after removing the subsidy, prices better reflect production costs and market value—a shift that guides manufacturers to adjust production to real consumer demand and optimizes resource allocation in the mobile industry. For key market participants like Apple and Huawei, subsidy-free price signals encourage technological innovation and quality improvement, thereby helping them retain market share and fostering healthy competition and long-term industry development. Additionally, from a policy evaluation perspective, this scenario provides a critical benchmark for assessing subsidy effectiveness, as comparing optimal utility and consumption structures across baseline, subsidy-withdrawal, and purchase restriction relaxation scenarios shows that subsidies essentially act as an income transfer and demand adjustment tool.

6. Limitations & Contributions

6.1 Limitations

This study has several limitations, as follows:

Firstly, it relies on a representative consumer with a fixed income and only two selected smartphone models, thereby ignoring the heterogeneity of income, preferences, and brand loyalty, etc., in the real world. Future studies could incorporate consumer heterogeneity across income levels, preferences, as well as age groups. Moreover, it's highly suggested to expand the product set to include additional smartphones to increase realism and generalizability.

Secondly, the policy shock considered in this essay is stylized and simplified, assuming it is immediate, while also omitting adjustment costs, behavioral biases, and real-world measures. By adopting dynamic analyses that account for time-dependent adjustments, behavioral responses, and market frictions, future research can better approximate real-world conditions.

Ultimately, the existing framework neglects firm-side reactions. Removing subsidies or purchase limits may induce firms to adjust their pricing strategies, product availability, as well as promotional strategies, which could potentially impact consumer choices. Future studies that extend the analysis into a two-sided model with firm optimization would provide a more complete policy evaluation.

6.2 Contributions

This study makes significant contributions to both the real world and the literature in several ways.

From a theoretical perspective, this study extends the application of the CES utility framework to a high-tech consumer market, providing a micro approach to analyze substitution between Apple and Huawei smartphones under policy constraints. By modelling three regulatory scenarios and comparing optimal bundles and utilities, the paper contributes to the literature on consumer choice, substitution elasticity, and distortions induced by policies. Moreover, it also complements previous work that applies the CES preferences mainly in industrial contexts, extending the use to analyzing markets with differentiated products and government intervention.

In practical terms, this essay delivers insights of direct relevance to policymakers, firms, and consumers in China. For policymakers, comparing welfare and consumption choices under subsidy removal and purchase limit relaxation provides governments with insights into potential efficiency losses or welfare gains when adjusting digital product consumption policies. For firms, this essay illustrates how changes in substitution elasticity will shift market shares, assisting strategic pricing and product planning. While for consumers, the study clarifies how regulatory constraints shape attainable bundles and effective costs, thereby improving the understanding of how policy interventions affect real purchasing behaviors.

7. Conclusion

This essay examines how government subsidies and purchase limits jointly shape consumer choices between Apple iPhone 17 and Huawei Mate 70 within a CES preference framework. The analysis establishes two key findings regarding the effectiveness of China's digital appliance subsidy program.

Firstly, the initial inclusion of a quantitative consumption ceiling introduces an unavoidable inefficiency. Although purchase limits will not change the monetary prices, they prevent consumers from achieving their optimum by imposing an implicit cost. As a result, attainable utility in the baseline scenario remains constrained, whereas removing the purchase limit enables consumers to fully express brand preferences.

Secondly, the efficiency of subsidies depends critically on the magnitude of substitutability. After removing the purchase limit, a higher ρ significantly boosts consumer responsiveness to relative prices, leading to a strong shift toward the more preferred brand. Under conditions of high substitution, the relative price advantage created by the uniform subsidy disproportionately amplifies demand for the already preferred product (Apple's iPhone), leading to a pronounced concentration of market share. Such dynamics highlight a potential issue: subsidies intended to support the entire market may instead reinforce the market dominance of the brand that consumers favor the most.

Overall, our findings underscore that policy design will not only affect the financial perspective but also how it interacts with the underlying preference structure. To optimize the policy's impact, it is highly recommended for policymakers to consider variable subsidy rates or employ moderate limits to prevent excessive market concentration under high substitutability. Alternatively, tiered subsidy rates or relevant subsidized models may mitigate unintended biases. It is quite challenging to balance the potential bias and social welfare, yet our essay provides insights into such a case, offering a clear framework for designing interventions that are both effective and equitable.

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